Methodological Approaches in the Study of Recent Mathematics: Mathematical Philosophy and Mathematical Practice

Abstracts

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(Universität Konstanz)

Contents

Abstracts

The Entanglement of Set Theory and Infinitary Model Theory

18 Sep
9:30
Invited

John Baldwin

University of Illinois at Chicago

We argued (2016 Aberdeen Symposium on Set Theoretic Pluralism) that the late 20th century paradigm shift in model theory largely freed *first order* model theory from dependence on axiomatic set theory beyond ZFC. We also asserted a serious entanglement between the model theory of infinitary logic and extensions of ZFC (see `http://homepages.math.uic.edu/~jbaldwin/pub/aber1.pdf`, `http://homepages.math.uic.edu/~jbaldwin/pub/aber3.pdf` and Baldwin, 2018). We explore the interaction of methodologies of the two subjects and differing intuitions that arise about the world of sets. This entanglement is expressed in several ways.

1) The large and small in model theory: Most results in mathematics outside of set theory are either about structures of size at most the continuum or are cardinal independent. Recent results in the study of infinitary logic have produced a dichotomy: some Hanf numbers, e.g., $L_{\omega_1,\omega}$-characterization of cardinals and related existence of maximal models, and the amalgamation spectrum (Hjorth, 2002; Baldwin, Koerwien, and Laskowski, 2016; Baldwin and Souldatos, 2017) are computable (say $\beth_{2^{\aleph_0}}$). Others are provably large cardinal properties. Others (some amalgamation issues, tameness, eventual categoricity, maximality) require large cardinals Boney, 2014; Boney and Unger, 2017; S. Shelah, 2013; S. Shelah and Vasey, 2018; Baldwin and S. Shelah, 2018. A Hanf number is essentially a compactness property. And these properties are 'algebraic', i.e.'theory building' versus 'Erdos-style' (combinatorial mathematics) Harris, 2015, page 192, fn 24 page 364. Can we find a precise meaning for 'algebraic' that explains these gaps? What do they assert about the eventual uniformity of the universe?

2) Set theoretic axioms and infinitary model theory: Maddy's *second philosopher* 'sees fit to adjudicate the methodological questions of mathematics - ... by assessing the effectiveness of the method ... as means towards the goal of a particular stretch of mathematics' (P. Maddy, 2007, 359). We look at two instances: the conflicting effect of Martin's axiom and $2^\kappa < 2^{\kappa^+}$ (weak GCH) on the study of infinitary amalgamation and categoricity. Can model theory provide an 'extrinsic justification' (in Gödel's sense) for set theoretic axioms?

Recent investigations into the model theory of almost ω-stable abstract elementary classes and whether $\aleph_1$-categoricity of sentences in $L_{\omega_1,\omega}$ is absolute have used iterated elementary embeddings of models of set theory to change consistency proofs of model theoretic propositions into proofs in ZFC (Baldwin, Larson, and S. Shelah, 2015; Baldwin, Laskowski, and S. Shelah, 2016; Baldwin and Laskowski, 2018. Does Maddy's P. Maddy, 2018 characterization of the role of ZFC as *metamathematical corral*, a place for freely exploring the relationship among various foundations, extend from the metamathematics of set theory itself and set theoretic topology to the foundations of model theory?

3) Compactness versus Incompactness: Cummings Cummings, 2018 provides six examples in each direction of the dichotomy. The obviously model theoretically significant ones are singular cardinal compactness and its obstacles Magidor and S. Shelah, 1994; Saharon Shelah, 1975, locally free but not free algebras. In Baldwin, 2018 I exemplify the paradigm shift in model theory as reversing the question from the set theorist's, 'For which *cardinals* (κ, λ) does T have a (κ, λ) model for all *theories* T?' to the *model theorist's*, 'For which *theories* T does T have a (κ, λ) model for all for all *cardinals* (κ, λ) for all *cardinals* ?' Now I seek a specific notions of compactness for which such a question reversal makes sense for infinitary logic. Can one give one find a notion of free to make precise the question, 'For which infinitary theories does locally free imply free'?

These three areas present 3 lines of analysis of the interacting methodologies of model theory and set theory.

Literature:

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W. Boney and S. Unger. "Large cardinal axioms from tameness in AECs". In: *Proceedings of the American Mathematical Society* 145 (2017), pp. 4517–4532.

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P. Maddy. *Second Philosophy, A naturalistic method*. Oxford University Press, Oxford, 2007, pp. xii + 448.

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S. Shelah and S. Vasey. “Categoricity and multidimensional diagrams,” preprint. 2018.

Forcing and the Universe of Sets: Must We lose Insight?

21 Sep
9:30
Invited

Neil Barton

Universität Wien

A central area of current philosophical debate in the foundations of mathematics concerns whether or not there is a single, maximal, universe of set theory. Universists maintain that there is such a universe, while Multiversists argue that there are many universes, no one of which is ontologically privileged. Often forcing constructions that add subsets to models are cited as evidence in favour of the latter. This paper informs this debate by analysing ways the Universist might interpret this discourse that seems to necessitate the addition of subsets to V. We argue that despite the *prima facie* incoherence of such talk for the Universist, she nonetheless has reason to try and provide interpretation of this discourse. We analyse extant interpretations of such talk, and analyse various tradeoffs in naturality that might be made. We conclude that the Universist has promising options for interpreting different forcing constructions.

Idealized Agents in Set Theory

17 Sep
16:00
Invited

Merlin Carl

Universität Flensburg

Constructivism is an appealing approach to clarifying the nature of mathematical objects and the way to obtain information about them. In its usual versions, it is unable to accommodate transfinite objects like they appear in set theory. However, in quite a few places in the foundations and philosophy of mathematics one finds attempts to justify the axioms of set theory in terms of higher idealized agency.

We will exhibit some such approaches, along with the criticism held against them. Then, we will propose two ways of formalizing idealized agency and apply these formalization to obtain information on the range and boundaries of such approaches.

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20 Sep
11:15
Contributed

A Naturalistic Case in Favour of the Generic Multiverse with a Core

Matteo de Ceglie
Universität Salzburg

In this paper, I compare the Generic Multiverse with a core (henceforth, GM_H) with the classical set theory ZFC, making use of the principle MAXIMIZE introduced by Penelope Maddy, 1997. This principle states that, since the aim of set theory is to represent all the known mathematics within a single theory, it should *maximize* the range of available isomorphism types. This is particular important for mathematics, since isomorphisms make it is possible to import methods and results from a mathematical field to another. I argue that the classic set theory ZFC is *restrictive* over the GM_H, that is, the GM_H *strongly maximizes* over ZFC in the sense that it provides a wide range of isomorphism types that are not available in ZFC.

I briefly define the GM_H as the multiverse with a common core of truths, shared between all the universes of the multiverse. A universe in this multiverse is a model of a certain set of axioms of set theory (for example $ZFC + V = L$ or $ZF + AD$), while the core is a set of propositions satisfied in every universe of the multiverse. Obviously in the multiverse there is also the universe that satisfies only the propositions in the core (that is, the core has a model that is part of the multiverse). All the other universes are extensions of this core: they satisfy all that is true in it, and more. For example, if the core is the the intended model of ZFC, the multiverse includes a model of $ZFC + V = L$ and a model of ZFC+ “$0^\#$ exists” (see Steel, 2014).

The GM_H thus defined strongly maximizes over ZFC: there is no theory T extending ZFC that properly maximizes over the GM_H and the GM_H inconsistently maximizes over ZFC. This means that the GM_H provides structures that cannot be satisfied by ZFC, even if properly extended. To see this, assume that the core of the GM_H is ZF^- (set theory minus the Axiom of Foundation). From this core we can build a multiverse in which, among others, there is a universe for ZFC and a universe for $ZF + AD$. In this multiverse one can have *both* the Axiom of Choice (provided by ZFC) *and* a full Axiom of Determinacy (provided by $ZF + AD$). Determinacy and Choice are actually incompatible, but they can coexist in the GM_H. Hence, the GM_H, unlike the intended model of ZFC, can include all the structures based on Determinacy. That is, the GM_H provides a new isomorphism type, i.e. it proves the existence of a structure that is not isomorphic to anything in ZFC.

Furthermore, the GM_H also provides what Maddy calls a *fair interpretation* of ZFC, i.e. the GM_H validates all the axioms of ZFC (this is because ZFC is part of the multiverse) and one can build natural models, inner models, and truncations of proper class of inner models at inaccessible levels of ZFC in the GM_H.

I conclude that, assuming MAXIMIZE, the GM_H is better justified than ZFC, since it provides more isomorphism types and it can fairly interpret ZFC itself.

Literature:
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A Made-to-measure Data-driven Approach

20 Sep
14:30
Contributed

Nick de Hoog
Universität Konstanz

Our research community has bemoaned that empirical data on 'real' mathematics, its plurality of practices and research cultures are not readily available. This scarcity is a problem to the philosophers who want to work in an empirically informed manner. As Löwe (2016, p. 38) noticed, this forces the empirical-based philosophy of mathematics, either into directing its attention to questions for which there is *off-the-rack* data available, or engaging into an interdisciplinary effort to generate bespoke data, or perhaps as a third option, investing much valuable time into generating the data yourself. In this talk, I want to discuss two sources that are easily accessible and show that these can be turned into a source of *made-to-measure* data supplementing various research questions.

As part of an effort to make more precise the notion of national (proving-)style in mathematics we used the pre-print-server arXiv.org and the database of the Mathematics Genealogy Project as keystone of our empirical study. Data from these sources is readily available and, once processed and enriched, resulted in a comprehensive database with information about articles and dissertations, as well as a large corpus of mathematical texts and proofs. In our particular study we used this information to uncover features of collaboration networks and to compare the properties of the products associated with two social relations, i.e., that of co-authorship and of supervisor-student.

By comparing the properties of the articles and dissertations inferences could also be made on a wide variety of themes that go beyond our specific topic, but may well suit others researchers in the field. In this talk, a few examples of such inferences are given. Although cleaning and enriching the data takes some effort, the result may offer a source of worthwhile made-to-measure data, serving as supplement for scholars of mathematical practices.

Literature:
Löwe B. (2016) Philosophy or Not? The Study of Cultures and Practices of Mathematics. In: Ju S., Löwe B., Müller T., Xie Y. (eds) *Cultures of Mathematics and Logic. Selected Papers from the Conference in Guangzhou, China, November 9-12, 2012*. Birkhäuser, Cham.

Generic Large Cardinals as Axioms

Sep 17
14:30
Contributed

Monroe Eskew
Universität Wien

We examine Matt Foreman's proposal to augment ZFC with generic large cardinal axioms, which are able to settle many classical independent questions such as the continuum hypothesis. The proposal runs into trouble with the mutual inconsistency phenomenon, which we prove is much more extensive than originally conjectured. We argue against Foreman's claim that generic large cardinals have the same justificatory status as conventional large cardinals. Finally, we propose a place for these axioms within a multiverse view.

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17 Sep
10:30
Invited

How to Study the Evolution of Set-theoretic Practices? Some Methodological Considerations

José Ferreirós

Universidad de Sevilla

No abstract available.

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20 Sep
9:30
Invited

The Stable Core

Victoria Gitman

City University of New York

Starting with Gödel's construction of L, set-theorists have succeeded in building increasingly more sophisticated canonical inner models, whose bottom-up construction according to absolute rules gives them regularity properties such as the GCH and $\square$, and makes them unaffected by forcing. An unrealized goal of the inner model program has been to construct a canonical inner model that is close to the universe in the presence of very large large cardinals. An inner model can be considered close to V for a number of reasons; it can satisfy a form of covering, it can agree about large cardinals, or V can be its class forcing extension.

Another inner model introduced by Gödel, the collection HOD of all hereditarily ordinals definable sets, does not structurally resemble the canonical inner models and can be easily modified by forcing because any set (or class) can be made ordinal definable in a forcing extension using coding. But HOD is consistent with all known large cardinals and it is conjectured that in the presence of very large large cardinals, HOD will be close to V. In trying to understand how close HOD can be to the universe V, Friedman introduced the ordinal definable *stability predicate* S and showed that V is a class forcing extension of the structure (HOD, S). The stability predicate S codes elementarity relations between initial segments H_α of V.

Indeed, V is already a forcing extension of $(L[S], S)$, a structure which Friedman called the *Stable Core* of V. Friedman showed that consistently $L[S]$ can be smaller than HOD, but most other properties of the Stable Core, such as whether it had regularity properties and whether it was compatible with large cardinals remained unknown. In a joint work with Friedman and Müller, we establish some further properties that are consistent with the Stable Core. We show that the Stable Core of $L[U]$ is $L[U]$, so the Stable Core can have measurable cardinals. We use coding techniques to show that $L[U]$ has a forcing extension with a measurable cardinal which is not even weakly compact in the Stable Core, so that measurable cardinals need not be downward absolute to the Stable Core. Using coding, we also show that the GCH can fail at all regular cardinals in the Stable Core. Our work leaves many open questions concerning the structure of the Stable Core in the presence of larger large cardinals.

Set-theoretic Independence of Existence of some Local Hidden Variable Models in the Foundations of Quantum Mechanics

18 Sep
14:30
Contributed

Michał Godziszewski

Uniwersytet Warszawski

In 1982 I. Pitowsky gave a construction of local hidden variable models (i.e. descirptions of quantum systems that provide deterministic predictions and are satisfied for spatially separated observables) for the so-called spin-1/2 (and spin-1) particles in quantum mechanics. Specifically, Pitowsky's main result was that under the assumption of the Continuum Hypothesis there exists a spin-1/2 function.

The function constructed in the proof of the theorem is non-measurable, making Pitowsky's model not directly subject to famous Bell's theorem, stating that under certain assumptions local hidden variable models are impossible. Since the construction uses CH, the natural question is whether the existence of Pitowsky's functions is provable in ZFC.

In 2012, I. Farah and M. Magidor demonstrated that it is actually independent. Namely they proved that:

1. if there exists a σ-additive extension of the Lebesgue measure to the power-set of the reals (i.e. if the large cardinal axiom known as 'the continuum is a real-valued measurable cardinal' holds), then Pitowsky's models do not exist, and

2. Pitowsky's functions do not exist in the random real model.

The second result thus shows that the non-existence of Pitowsky's functions is relatively consistent with ZFC. The proofs of these theorem rely on the results by H. Friedman and D.H. Fremlin, specifying that under the assumptions of (1) and (2) Pitowsky's functions have to be Borel-measurable.

Farah and Magidor indicated that the results above mean that physics might serve as a source of arguments for or against adopting specific new axioms of set theory. In 2016, J.Kellner argued against that, claiming that Pitowsky's analysis and his notion of hidden variables are just in fact just super-determinism (and accordingly physically not relevant), and hence without any possible influence on the adoption of particular principles as axioms.

During the talk I will try to sketch Pitowsky's construction, explain the proofs of the Farah-Magidor theorems, and relate them to the above-mentioned philosophical debate on in the quantum foundations and philosophy of set theory, arguing against Kellner's conclusion.

20 Sep
13:30
Contributed

Embedding Posets into the Set-Generic Multiverse

Miha Habič

České vysoké učení technické/Univerzita Karlova

In 1976 Mostowski showed that, given a countable transitive model M of GBC, any finite poset may be embedded into the structure of extensions of M with the same first-order part. The embedding he obtained also has the interesting property that it preserves *nonamalgamability*, i.e. the nonexistence of upper bounds. The key (but not the only) ingredient of Mostowski's proof is the construction of a nonamalgamable pair of models of GBC.

I will present the results of a recent project with Hamkins, Klausner, Verner, and Williams, in which we discovered that the nonamalgamability phenomenon is widespread in several set-theoretic contexts. In particular, if we consider a countable model of ZFC and the set-generic multiverse of all of its forcing extensions, we showed that it too can accommodate any desired finite poset (and even some infinite ones) while respecting amalgamability and, with some more work, also meets. The forcing extensions themselves can be chosen to come from a wide variety of forcing notions: they might all be obtained by adding Cohen reals, or some might collapse many cardinals while others remain benign. Similar results hold for the class-generic multiverse over a model of a sufficiently strong second-order set theory, such as GBC with Elementary Class Choice.

These results point to a striking richness of the set-theoretic multiverse in its many forms and suggest parallels between it and the structure of the Turing degrees arising in computability theory. This viewpoint is reinforced by the fact that a number of computability-theoretic methods were used in obtaining the above results, and that these methods remain fruitful in exploring the set-theoretic multi- verse. For example, one can show that there is an exact pairs of forcing extensions, i.e. a pair of extensions of a common ground model with no greatest lower bound, using an argument similar to the one showing the same result for Turing degrees. This suggests that there should be a deeper connection between the established rich structure of the Turing degrees and the structure of forcing extensions of a given model.

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17 Sep
13:30
Contributed

Large Scale Quantitative Investigation of Mathematical Publications

Mikkel Willum Johansen

Københavns Universitet

Three decades ago Reuben Hirsh famously pointed out that mathematics has a front and a back; there is qualitative differences between the way the dish is prepared and the way it is served to the public (Hersh 1991). A recent qualitative interview study I conducted in collaboration with Morten Misfeldt partly confirmed this conclusion; Although the mathematicians we interviewed relied heavily on informal modes of reasoning such as diagrams and figures in their work practice they tended to downplay or omit figural representations in their published papers (Johansen and Misfeldt 2016). The mathematicians we interviewed all accepted the importance of rigid (formalizable) proofs, but some also expressed discontent with the current publication norm that calls for restriction in the use of diagrams.

From a casual point of view, it seems that the norm is in fact in a process of change. The formalistic claim that figures and diagrams are superfluous has been contested by philosophers of mathematics (e.g. Brown 1999, Giaquiont 2007), and when one leaf through mathematics journals and textbooks, one gets the impression that diagrams and figures are being used more frequently. That however is merely an impression. We do not have solid empirical evidence tracking changes in the use of diagrams and figures in mathematics texts, and worse still: we do not have a solid empirical method making such a tracking possible. For that reason, we do not in fact know if the norms governing the publication practice in mathematics is changing.

In this talk I will present the results of joint work conducted by Morten Misfeldt (Aalborg University), Josefine Lomholt Pallavicini (University of Copenhagen) and myself. Together, we developed a classification scheme that makes it possible to distinguish between the different types of diagrams used in mathematics based on the cognitive support offered by diagrams (Johansen et al). The overall goal of the classification scheme is to facilitate large-scale quantitative investigations of the norms and values expressed in the publication style of mathematics, as well as trends in the kinds of cognitive support used in mathematics. In the talk I will present the difficulties we faced in the development of the research tool, and I will present preliminary results showing trends and changes in the publication practice of Annals of Mathematics during the last century.

Literature:
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Johansen, M.W. and Misfeldt, M. (2016): An empirical approach to the mathematical values of problem choice and argumentation. In Larvor, B. (ed.): *Mathematical cultures: The London meetings 2012-2014*, pp. 259-269. Switzerland: Springer.
Brown, J. R. (1999): *Philosophy of mathematics, an introduction to a world of proofs and pictures. Philosophical Issues in Science*. London: Routledge.
Giaquinto, M.(2007): *Visual Thinking in mathematics, an epistemological study*. New York: Oxford University Press.
Johansen, M.W., Misfeldt, M. and Pallavicini, J.L. (in press): *A Typology of Mathematical Diagrams*.

Interviews with Set Theorists - Different Views on Forcing

21 Sep
12:15
Invited

Deborah Kant
Humboldt Universität zu Berlin

In this talk, we first describe the methodology of empirical research based on interviews with set theorists. The method comprises three main steps: Interviews, transcription, and analysis. In particular, we consider questions such as ‘How should the interview questions be formulated?’, ‘How do we guarantee the anonymity of the interview partners?’ and ‘How can we compare answers by different interview partners?’.

Second, we present specific views on forcing and show that some of them differ significantly. Taking into account other interpretations of forcing in the philosophical literature, we attempt a first answer to the question where empirical research can have its place in philosophy of set theory.

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19 Sep
9:30
Invited

Algebra-valued Models of Set Theory and Their Logics

Benedikt Löwe

Universiteit van Amsterdam/Universität Hamburg

The construction of Boolean-valued models of set theory goes back to work by Solovay and Vopenka in the 1960s and is closely related to the theory of forcing: given a model of set theory V and a Boolean algebra B, we construct a set of names, a forcing language, and an assignment of truth values in B to the sentences of the forcing language. This construction generalises to other truth value algebras and was, e.g., used to get models of constructive set theory: using a Heyting algebra in which *tertium non datur* does not hold, one can obtain a Heyting-valued model of set theory in which *tertium non datur* does not hold. The underlying intuition is that the logical properties of the algebra used in the construction should be reflected in the logic of the algebra-valued model of set theory; however, that is not true in general. We call such a model "loyal" if the logic of the algebra is the logic of the algebra-valued model of set theory and discuss cases in which the algebra-valued models of set theory are illoyal. This talk reports on joint work with Lorenzo Galeotti, Robert Passmann, and Sourav Tarafder.

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18 Sep
16:00
Invited

Inspirations from Online Collaborative Mathematics into Automated Reasoning

Alison Pease

University of Dundee

New crowdsourcing technology is enabling new forms of collaboration in mathematical research, extending the power and limits of individuals. We look at what we can learn from these new forms of collaboration: online searchable records providing insights into what mathematicians talk about, how they explain things, what they value and what the patterns of communication might be. We discuss the role of the machine in this new context: whether it is to enable communication, to perform some of the "drudge" tasks, or to contribute in a creative way to the production of mathematics. We give three examples of software we have developed which build on these insights and, we hope, will help us to close the gap between automated and human reasoning. These are all based on the role of the 'example' in mathematics: how examples supporting or refuting a given conjecture can trigger conflict, ambiguity, understanding and change.

The Usage of Frameworks from Philosophy of Science in Philosophy of Mathematics

18 Sep
11:15
Contributed

Deniz Sarikaya

Universität Hamburg

In this talk I want to present some work in progress concerning the question whether we can use frameworks from Philosophy of Science to describe mathematical research. To be more precise: I want to discuss the *schema of normal phases versus revolutionary phases* introduced by Thomas Kuhn and the *semantic view of theories*. The first idea is rather advanced, the second is still in the beginning and I am seeking for feedback about the design of the project.

Ad revolutions: There is strong historical evidence that mathematical terminology and symbols are changing their meaning over time. The most famous example is the rational reconstruction of the theory of polyhedrons by I. Lakatos (1976). Is the change of the meaning of the term 'function' before and after Leibniz comparable to the change of the meaning of 'planet' before and after the heliocentric revolution? Do these meaning shifts yield to incommensurability as discussed in Kuhn's *The Structure of Scientific Revolutions* (1962)? I want to discuss meaning shifts in context of the process of formalization esp. in the realm of mathematical logic. This can mean friction between the formal theory and its quasi-formal counterpart or friction between the formal theories f.i. between ZFC and NF as different axiomatizations of the notion of set. This includes the following questions: Is there a difference between the reconstruction of mathematics within ZFC and the reconstructed realms? Do mathematicians of different foundational schools (like intuitionists vs. classical mathematicians) follow different paradigms? The *semantic view of theory* states that a theory can be identified with a collection of its models, rather than a set of its true statements. I want to investigate in how far different axiomatic systems in the context of set theory can be ordered into a theory net. I will investigate if we can treat an extension of ZFC in the context of the Gödel-program like a *specification* of a scientific theory. Furthermore, I want to address different conceptual notions of set via the idea of *intended models*. The key idea is to use frameworks which analyse the structure between different models and theories in philosophy of science. This transfer has been made to other (non-scientific) theories with some success, like for classical utilitarism (Gähde 1988).

Literature:

Gähde, Ulrich (1988): Changes in the concept of utility in classical utilitaranism. In: *Philosophie des Rechts, der Politik und der Gesellschaft : Akten des 12. Internationalen Wittgenstein Symposiums* / Ed. Ota Weinberger et al - Wien: Hölder-Pichler-Tempsky, pp. 82-84.

Lakatos, Imre (1976): *Proofs and refutations: The logic of mathematical discovery*. Cambridge and New York: Cambridge University Press.

21 Sep
11:15
Contributed

On the Role of Outer Models in the Methodological Maxim of Maximization

Jeffrey R. Schatz

University of California, Irvine

Much of the contemporary work on the methodology of axiom selection in set theory has focused on the maxim of maximization. While there have been a number of formal explications of this maxim, one of the most fruitful has been Maddy's approach: an extension of ZFC T maximizes over another T' if T has a fair interpretation of T', and also proves that this interpretation lacks some isomorphism type. While Maddy defines a fair interpretation as a particular sort of inner model interpretation of the theory T', one might naturally consider also outer model "interpretations" of theories in this approach to maximization. Given the increasing understanding in both mathematical and philosophical communities of the relationship between ground models and their forcing extensions, such an extension of Maddy's approach holds the potential of permitting a more detailed and fine-grained analysis of how theories can maximize over one another. Furthermore, given the contemporary debate between various strengthenings of $ZFC+LCs$, in particular through the $Ult(L)$ program and through forcing axioms, the philosophical of study of maximization notions promises to be an area of significant and productive inter-disciplinary work between philosophers of mathematics and working set theorists.

In this talk, we begin by briefly considering the historical development of the methodological question of how to decide between incompatible extensions of ZFC, focusing in particular on how the development of Cohen's forcing method brought new importance to this question. We will then present and consider a friendly amendment to Maddy's notion of maximize, allowing certain types of forcing extensions of inner models to also serve as fair interpretations. We argue that this new definition of fair interpretations requires an additional "covering" requirement be added to the definition of maximization, which properly extends the motivations behind Maddy's notion of maximization to the new case of outer models. We then show that despite presenting a seemingly alternative approach to maximization, this friendly amendment is in fact functionally equivalent to Maddy's original definition: in particular, T maximizes over T' in the extended definition if and only if it already maximized over T' in the original definition. We then examine a few potential implications of this result for contemporary debates between alternative axiom systems. Finally, we conclude by considering the implications of these results for the broader question of whether outer models should be considered genuine intended models of strong theories of sets, outlining promising future work towards a better understanding of the precise role of outer models in questions of axiom selection.

Semiotic Analysis of Mathematical Texts (A Hands-on Introduction with Applications to P. Cohen's Presentations of Forcing)

20 Sep
16:00
Invited

Roy Wagner
ETH Zürich

No abstract available.

What Could Set-Theoretic Explanations in Science Be?

18 Sep
13:30
Contributed

Krzysztof Wójtowicz
Uniwersytet Warszawski

The talk examines the possible impact of investigations within set theory on the ongoing discussion concerning mathematical explanations in science (especially: physics) – and *vice versa*.

The thesis, that there are non-causal, genuinely mathematical explanations in science has a strong support. According to it, phenomena (in physics, biology, chemistry, brain sciences etc.) are explained by referring to the truths of mathematics (expressing abstract properties of the system), not by identifying the causal mechanisms and the detailed scenarios. Usually such explanations make use of theorems arising from the "everyday mathematical practice" – like differential equation, (topological) graph theory, probability theory – and many others.

Set theoretic results seem to be too abstract in such contexts. But independence phenomena within "ordinary mathematics" are well known – also with respect to extensions of ZFC (e.g. Friedman's results). There are also examples of independent sentences having a physical interpretation – for instance concerning models for relativity theory (da Costa, Doria), but not only. They provide a strong link between the seemingly "too abstract" set-theoretic principles and the body of mathematical and scientific practice. Such results are of particular interest, as two important discussions, concerning:

1. mathematical explanations in science;

2. the practice and philosophy of set theory (and its relationship to "ordinary mathematical practice")

become entangled and mutually inspiring – in a way interesting for both the philosophers of set theory and philosophers of science. The problem becomes especially interesting – or even pressing – for the mathematical realist, in particular holding the multiverse view (in the full-blooded Platonism spirit).

One of much-discussed approaches to mathematical explanations is treating mathematical theorems as a kind of modal constraints. But in this case the status of the background assumptions becomes crucial. Contemporary set theorists are used to work with diverse models, suited to the mathematical problem in question, so it is natural to ask about their metaphysical and epistemological status in this more general setting. In particular, the question, whether the independence results obtained by the forcing method are philosophically innocuous to science, becomes acute.

The notion of mathematical explanation in science is well established, but a more specific notion of set-theoretic explanations is also promising. Set theory is focused on studying models, and in particular the (in a sense: meta-theory dependent) properties of them. And properties of models – not only of the physical systems in question – would be essential for this type of explanations. The notion might prove interesting not only from the point of view of broad philosophical discussion, but also for the set-theoretic investigations. Studying diverse models for set theory might therefore turn out to be not only interesting *per se*, but have a very direct bearing on crucial philosophical questions, concerning the applicability of mathematics and the way it enters scientific theorizing. And the problem of the possible explanatory role of set-theoretic results could be addressed in a precise way, in the spirit of formal (mathematical) philosophy.
